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A Detailed Explanation of Solenoid Force

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***Abstract*— This article explains the manner in which the force of a solenoid varies with coil diameter, coil length, wire gauge, supply voltage, packing density, and the number of turns. Particular attention is given to explaining how force varies with the number of turns, as the author has found that solenoid behavior in this regard is often non-intuitive and surprising for engineers who have only been exposed to basic magnetic field expressions for coils.**

***Index Terms*—solenoid, force**

I. Introduction

A primary motivation for this article was the desire for students in an embedded systems course to have an understanding of solenoids sufficient to answer questions such as these:

1. You have a base solenoid design but would like to obtain more force. What parameters can you vary in order to do so?
2. You have more force than you need from a particular solenoid. What can you do to reduce it and save on power in the process?

On the surface these would seem to be questions that should require only a rudimentary understanding of solenoids, but an examination of basic literature on solenoids shows that to not be the case. For example, undergraduate textbooks on electromagnetism universally provide the following approximation to the magnetic field of a long cylindrical coil (e.g., [1]):

$$B = \frac{\mu NI}{l} \tag{1}$$

Here N/l is the density of turns, I is the current through the coil, and μ is the permeability of the core. The analysis of the field for cylindrical coils does not go much beyond that, even in textbooks oriented towards engineering students. A novice is likely to look at this equation and conclude that one can increase the magnetic field, and thus the force of the solenoid, by adding turns. A problem with that conclusion is that practical circuits rarely drive coils with constant current; they almost always apply constant voltage.

As a second example, standard texts are likely to show the following formulations for the energy of an inductor coil [1]:

$$W = \frac{1}{2} L I^2 \tag{2}$$

$$w = \frac{B^2}{2\mu} \tag{3}$$

Where L is the inductance, (2) is total energy, and (3) is energy density, which must be integrated over an enclosing volume to get energy. If we substitute (1) into (3) we see that both of these formula are again expressed in terms of current.

It is natural at this juncture to attempt to make use of Ohm's law and substitute $V=I/R$ into either of these to get an expression in terms of voltage. However, these devices are inductive, and so there is also the reactive component of impedance to consider. This cannot be ignored even if you're driving a coil with a DC waveform, because determining an inductor's reaction to a sudden change of voltage requires consideration of reactance. It may be tempting at this point to focus on the steady state and dismiss the phase shift between voltage and current due to reactance and make the following substitution:

$$|I| = \frac{|V|}{|Z|} = \frac{|V|}{2\pi f L} \tag{4}$$

where f is frequency. If we use radial frequency, $\omega=2\pi f$, drop the magnitude symbol, and substitute into (2) we obtain:

$$W = \frac{V^2}{2\omega^2 L} \tag{5}$$

This same expression can be obtained by multiplying the average reactive power in an inductor by one radial time period, $1/\omega$, in order to obtain the energy over one cycle:

$$P_{ave} = V_{rms} I_{rms} = \frac{V}{\sqrt{2}} \frac{I}{\sqrt{2}} = \frac{V^2}{2\omega L} \tag{6}$$

$$W = P_{ave} T = P_{ave} \frac{1}{\omega} = \frac{V^2}{2\omega^2 L} \tag{7}$$

In this case, we can simplify the development by normalizing, at least temporarily, to a radial frequency of 1 rad/sec, leading to:

$$W = \frac{V^2}{2L} \tag{8}$$

This and (2) exhibit symmetry to the corresponding formula for the power dissipated by a resistor. This also appears to give us what we are seeking: an expression for the energy in a coil for a constant voltage. Of course we assumed a steady-state AC voltage to get here, but perhaps that is acceptable since we know that a solenoid that works for AC can be made to work for DC. For readers that are wondering how a solenoid works for AC when the B field is constantly reversing, all you need to realize is that the force

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on the armature will be in the direction that reduces inductance. Thus, regardless of the direction of B, the armature will always be pulled into the coil.

Unfortunately, pursuing the formulation in (8) leads to a result that does not approximate real solenoid behavior because it is based on an unrealistic model of a coil. That analysis, along with an explanation of the mismatch, is given in Appendix A. For now it will prove more useful to pursue an approach that does work, starting from a realistic equivalent circuit that includes both an ideal inductance along with a series resistance to represent copper losses in the coil.

II. Equivalent Circuit

Fig. 1 shows an equivalent circuit of an ideal inductor (the coil of the solenoid), along with a series resistor representing the lumped resistive losses of the coil. The switch is thrown at time $t=0$, and we wish to determine $I(t)$ and $V(t)$ across the inductor. The product of these will give us power, and we integrate the power over time to get energy. Inductance will vary with the position of the solenoid armature. The positions of armature out and armature all the way in are the easiest to analyze because the inductance is easy to estimate in those positions. However, it is also instructive to differentiate energy with respect to armature position in order to get force, and we can then compare the force vs. position to force-stroke curves for real solenoids. This does, in fact, provide a justification for rejecting a model based on (8).

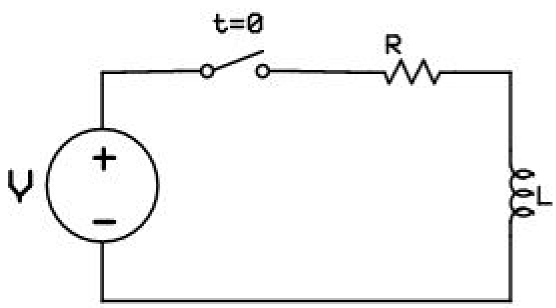

Figure 1. Equivalent Circuit

A detailed analysis of the time-varying current and inductor voltage can be found in any electric circuit textbook (e.g., [2]) and at many sites on the web (e.g., [3]). The transient response of the inductor is required here, and may be obtained using Laplace circuit analysis or via a straightforward solution to a first-order differential equation. Here we just make use of the pertinent results, rather than cover the details.

The voltage across, and current through, the inductor are:

$$V_L = Ve^{-\frac{t}{\tau}} \tag{9}$$

$$I_L = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right) \tag{10}$$

where $\tau = L/R$ is called the RL time constant. The waveforms are shown in Figs. 2 and 3. Note that the current builds gradually to V/R and the voltage spikes to V and then falls off gradually to 0. The circuit comes to within ≈37% of its final state in τ seconds.

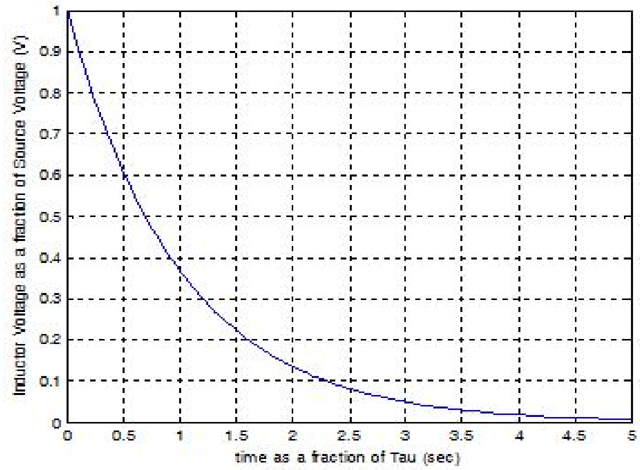

Figure 2. Voltage across the inductor

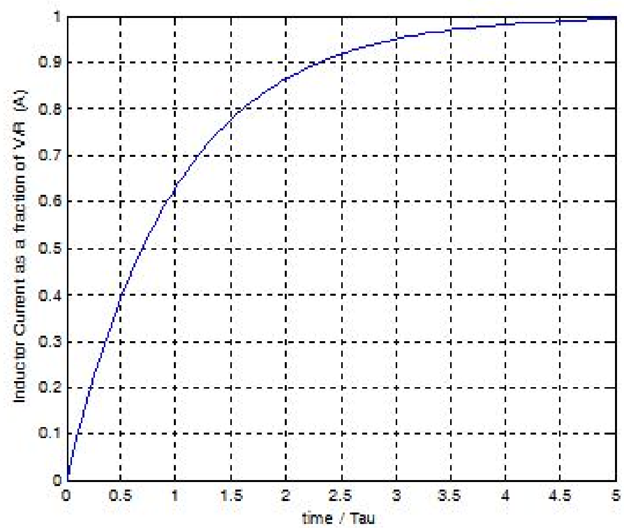

Figure 3, Current through the inductor

Multiplying voltage and current gives power:

$$P_L = I_L V_L = \frac{V^2}{R}\left(e^{-\frac{t}{\tau}} - e^{-\frac{2t}{\tau}}\right) \tag{11}$$

Integrating the power gives the energy required to bring the inductor up to its steady-state current:

$$\begin{aligned} W_L &= \int_0^\infty P_L dt \\ &= \int_0^\infty \frac{V^2}{R}\left(e^{-\frac{t}{\tau}} - e^{-\frac{2t}{\tau}}\right)dt \\ &= \frac{V^2}{R}\left[-\tau e^{-\frac{t}{\tau}} + \frac{\tau}{2}e^{-\frac{2t}{\tau}}\right]_0^\infty \\ &= \frac{V^2}{R}\left[\tau - \frac{\tau}{2}\right] = \frac{V^2}{R}\frac{\tau}{2} = \frac{V^2 L}{2R^2} \end{aligned} \tag{12}$$

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Like (8), this equation for energy gives us what we desire: an expression in terms of voltage instead of current. Note, however, that there are two significant differences: (a) it includes resistance, which is a more realistic model, and (b) inductance appears in the numerator instead of the denominator.

III. From Energy To Force

Force is the derivative of energy with respect to the position of the armature. For this we are going to need an expression of the variation of inductance with position, *L(x)*. We begin by defining a coordinate system, as shown in Fig. 4:

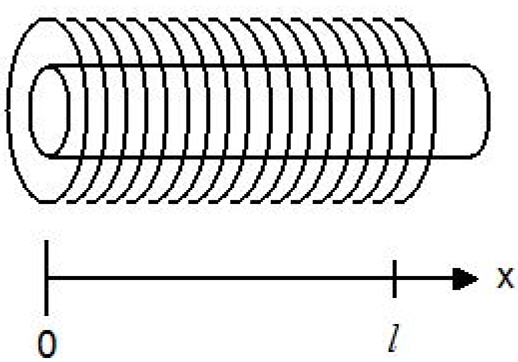

Figure 4. Armature coordinate system

At x=0 the armature is all the way inside the coil. At $x=l$ the armature is at the entry edge of the coil. For these two positions the inductance is easy to define using the standard approximation for the inductance of a coil:

$$L(0) = L_0 = \frac{\mu_r \mu_0 N^2 A}{l} \tag{13}$$

$$L(l) = L_l = \frac{L_0}{\mu_r} = \frac{\mu_0 N^2 A}{l} \tag{14}$$

It can be advantageous to define l as somewhat farther outside the coil in order for the armature to avoid the fringe of the B field, thereby improving the accuracy of (14). For the sake of simplicity, however, we use the length of the coil in (14) and provide an alternative way to make that adjustment below. *L(0)* is much larger than *L(l)* because μ_r, the relative permeability of the iron armature, is much larger than μ_0, the permeability of free space. *L(x)* will vary monotonically between these two extremes, and the precise shape of this variation with position depends on the construction and shape of the solenoid. A precise model of this variation requires development of a detailed spatial model (such as a finite element model) of the solenoid's magnetic field. For our purposes here, all we require is a reasonable approximation for a cylindrical solenoid, and for now will assume an exponential decay:

$$L(x) = L_0 e^{-\frac{\alpha}{l}x} \tag{15}$$

As required, this has the value L_0 at $x=0$, and we want to choose the parameter α such that $L(l)=L_0/\mu_r$:

$$\begin{aligned} L_0 e^{-\frac{\alpha}{l}l} &= \frac{L_0}{\mu_r} \\ e^{-\alpha} &= \frac{1}{\mu_r} \\ -\alpha &= \ln\left(\frac{1}{\mu_r}\right) \\ \alpha &= \ln(\mu_r) \end{aligned} \tag{16}$$

A value for α somewhat less than this, but larger than unity, will have the same effect as moving l outside the end of the coil. We can now derive the force, as follows:

$$E_L = \frac{V^2}{2R^2} L(x)$$

$$F = \frac{dE_L}{dx} = \frac{d}{dx}\left[\frac{V^2}{2R^2} L(x)\right] = \frac{V^2}{2R^2}\frac{d}{dx} L(x)$$

$$\frac{d}{dx} L(x) = L_0 \frac{d}{dx} e^{-\frac{\alpha}{l}x} = -L_0 \frac{\alpha}{l} e^{-\frac{\alpha}{l}x}$$

$$F(x) = -\frac{V^2}{2R^2}\frac{\alpha}{l} L_0 e^{-\frac{\alpha}{l}x} \tag{17}$$

At this point we make several substitutions in order to get the force in terms of coil design parameters. We start by computing R from the resistance per unit length of the wire, γ, and the total length of wire, which is computed from the number of turns times the average circumference of the turns:

$$R = 2\pi r_a N \gamma \tag{18}$$

It should be noted that r_a is not the same as the inside radius of the coil, which we shall call r_0. The difference between these is illustrated in Fig. 5.

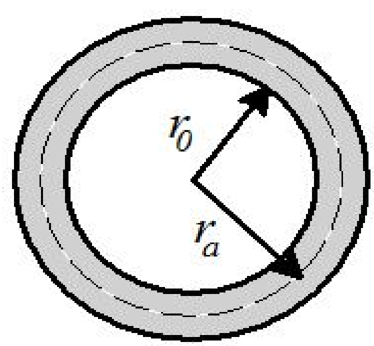

Figure 5. Coil radius vs. average turn radius

The parameter γ may be obtained from a table of wire gauges (e.g., [4], [5]), and is also computable from the resistivity of copper, ρ, and the cross-sectional area of the wire, a:

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$$\gamma = \frac{\rho}{a} \tag{19}$$

We also substitute into (17) the inductance at the stop position, L_0, along with the cross-sectional area of the coil opening, A:

$$L_0 = \frac{\mu_r \mu_0 N^2 A}{l} \tag{20}$$

$$A = \pi r_0^2 \tag{21}$$

Substituting (21) into (20), and substituting the result along with (18) into the last line of (17), results in:

$$\boxed{\begin{aligned} F &= \frac{-V^2 \mu_r \mu_0}{8\pi \gamma^2 l^2} \left(\frac{r_0}{r_a} \right)^2 \alpha e^{-\frac{\alpha}{l}x} \\ &= \frac{-V^2 \mu_r \mu_0}{8\pi \gamma^2 l^2} W_f \alpha e^{-\frac{\alpha}{l}x} \end{aligned}} \tag{22}$$

In (22) we have defined the squared ratio between r_0 and r_a as a new parameter, the *winding factor*, W_f. The winding factor is always less than unity, and represents a reduction on what might otherwise be considered the nominal force:

$$\begin{aligned} F &= F_{nom} W_f \\ W_f &= \left(\frac{r_0}{r_a} \right)^2 \end{aligned} \tag{23}$$

IV. Discussion

A comparison between the predictions of (22) and a commercial cylindrical solenoid are illustrated in Fig. 6. The simulated coil used the following parameters: l=27 mm, r_0 = 2.3 mm, r_a = 4.5 mm, AWG = 30, N=572. Both the simulated and actual coil resistances were 5.3 Ω. As can be seen, the simulated curve matches the high power curve for the solenoid quite well. At lower powers, the force of the actual solenoid drops off at a higher exponential rate. This could be simulated by substituting a higher power of x in the inductance formula of (15).

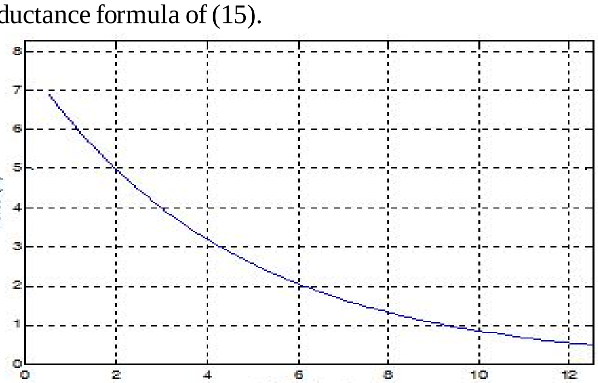

Figure 6a. Simulated Force

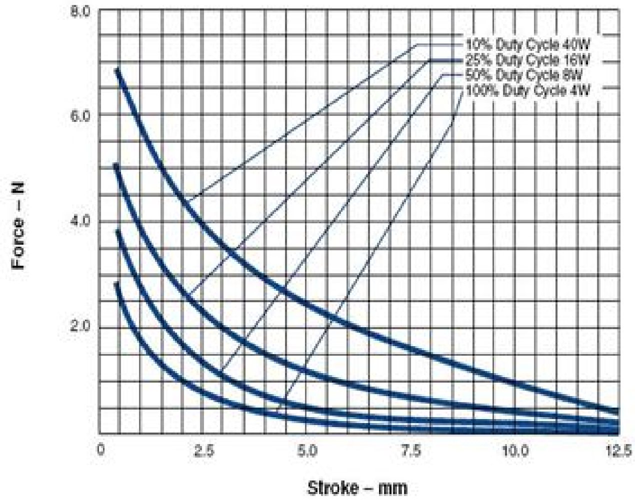

Figure 6b. Actual Force

It is worth noting that assuming a linear variation in inductance from *L(0)* to *L(l)* would result in a force that is constant vs. stroke. While there are some actual solenoids that approximate that, the more usual case is the exponential variation shown in Fig. 6. This is why we modeled the variation in inductance as an exponential decay, rather than linear, in (15). As stated earlier, the precise shape of *L(x)* depends on the geometric construction of the solenoid.

Notice also that N does not appear in (22) at all. An N^2 term shows up in the denominator as a part of the squared resistance, but N^2 also shows up in the numerator as part of the inductance, and the two effects thus cancel out. This means that while removing turns will increase the current, it will also decrease the magnetomotive force of the B field by the same factor. Another way to recognize this canceling effect is to look at the estimation of the magnetic field in (1). In that equation, removing turns decreases B, but it also increases I due to a proportional reduction in R, and the two effects cancel each other.

There is, however, another factor that must be considered. Eq. (22) does not account for the fact that r_a is actually a function of N. For a fixed length coil, increasing N will eventually increase r_a. But since r_0 remains unchanged, the ratio between the two is reduced. Since this ratio is always less than unity, this means that adding turns will gradually decrease the force. In order to examine the extent of this effect, we must substitute into (22) a model for the dependence of r_a on N. Fig. 7 shows a cross-section of wires coming around the solenoid core.

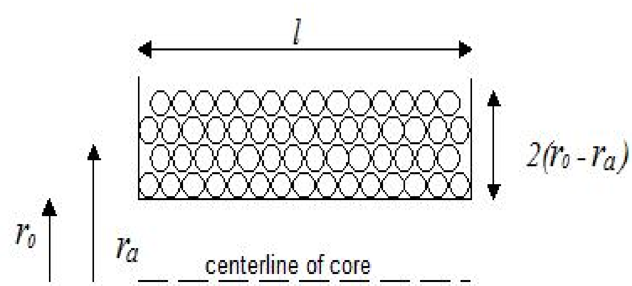

Figure 7. Turn packing

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The space available for these N turns is $2l(r_0-r_a)$. The area taken up by the wires is a proportion of this defined by the packing density, λ. The theoretical maximum packing density for the lattice arrangement shown above is $\pi/\sqrt{12} \approx 0.907$ [6]. If the wires are stacked in a grid instead of a lattice, the packing density is easily shown to be $\pi/4 \approx 0.785$. We can now express r_a as a function of N, as follows, where a is the cross-sectional area of the wire, available from an AWG table [4,5]:

$$\lambda A_{avail} = A_{wires}$$
$$\lambda 2(r_a - r_0)l = Na$$
$$r_a = \frac{a}{2\lambda l}N + r_0 \quad \lambda \le \frac{\pi}{\sqrt{12}} \tag{24}$$
$$= \beta N + r_0$$

For brevity, we have collapsed λ, l, and a into a single parameter, $\beta = a/(2\lambda l)$.

Substituting (24) into (22), and further simplifying by examining the force at x=0, we obtain:

$$\boxed{F_0 = \frac{-V^2\mu\alpha}{8\pi\gamma^2 l^2}\frac{r_0^2}{(\beta N + r_0)^2}} \tag{25}$$

This reveals a refinement on the winding factor in (23):

$$W_f = \frac{r_0^2}{(\beta N + r_0)^2} \tag{26}$$

When N is small, W_f will approach the squared ratio of r_0 to r_a used in (23). As N increases, the ratio becomes smaller. Thus, N does play a role in the force, but it is a secondary effect having to do with the increase in r_a, not the increase in resistance. The rate at which an increase in N will decrease the force depends on the value of β, which depends on the packing density, the wire gauge, and the length of the coil. In order to illustrate the nature of this variation, we now look at the effect of reducing N on the winding factor, the relative force (compared to nominal), and the relative power, for a coil with the following parameters[1]: l=36 mm, r_0 = 7 mm, AWG = 26, a=0.129 mm^2, N_{nom}=1305 [7].

Fig. 8 shows the variation in r_a along with the winding factor. From 100 to 1305 turns, the average turn radius increases linearly from 7.2 mm to 9.5 mm. Although the denominator is quadratic, in this case the shape of the winding factor in the region of interest is nearly linear. The reduction in winding factor will create a proportional reduction in force, as required by (22) and (25).

Fig. 9 shows the variation in relative force and relative steady-state power as a consequence of varying N. The relative force has the same shape as the winding factor above, except that it is made relative to the nominal force at N=1305. The relative power is inversely proportional to $R = 2\pi r_a N\gamma$, where r_a is a function of N. As an example, these curves predict that reducing the number of turns from 1305 to 400 will increase the force to 150% of nominal, but at the cost of an increase in power to 400% of nominal. A decrease of turns from 1305 to 700 will increase the force to 130% of nominal, at an increase in power to 210% of nominal.

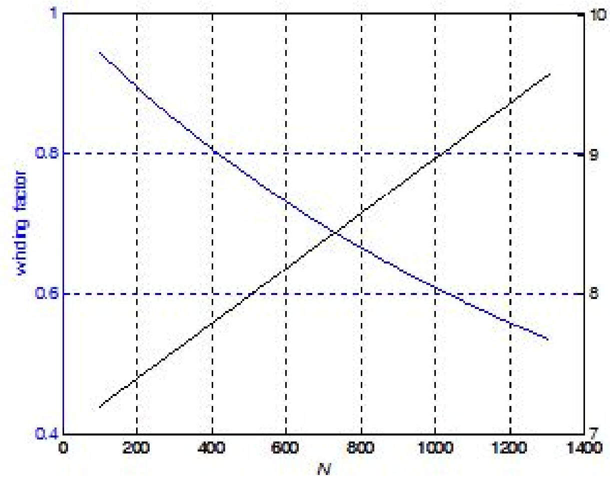

Figure 8. Winding factor and average turn radius vs. N

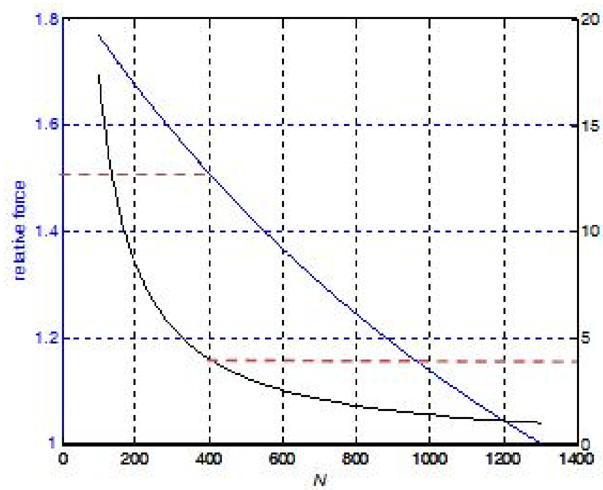

Figure 9. Relative force and power vs. N

Fig. 10 plots the ratio of relative force to relative power as an efficiency measure. At 400 turns, the efficiency is 38% what it is at 1305 turns. This shows that N should be treated as a secondary factor in designing a coil for a desired force. Referring to (22) and (25), other coil parameters that are important in the force calculation are the resistance per unit length of the wire, γ, and the total length of the coil, l.

As shown in Figs. 11 and 12, it is much more efficient to increase force by decreasing γ (decreasing wire gauge). These figures show the same quantities as Figs. 9 and 10, for a 400 turn coil with γ (and a) ranging from the values for AWG16 through AWG25. Note especially that unlike decreasing N, decreasing γ not only increases the force, it also increases efficiency.

Varying l offers another strategy for increasing force. Figs. 13 and 14 show these measures again for a 400 turn coil using AWG24 wire, with l varied from 25 to 50 mm. As with γ, decreasing l increases both force and efficiency. In this case, however, the relative force curve offers more steady, and modest, increases.

[1]Alvin G & Co, part number CLL-006, 26-1305

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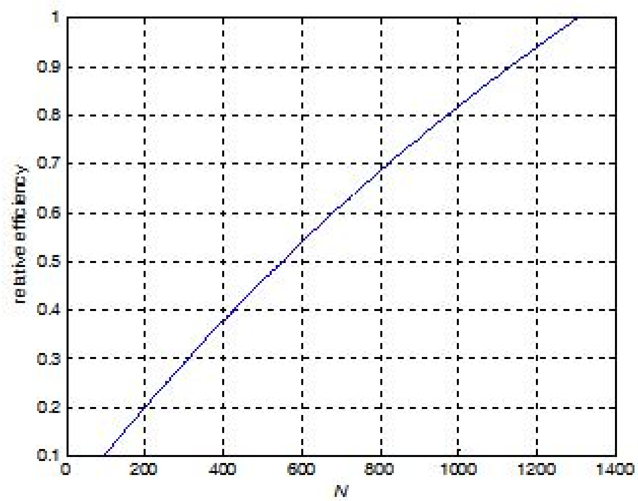

Figure 10. Relative efficiency vs. N

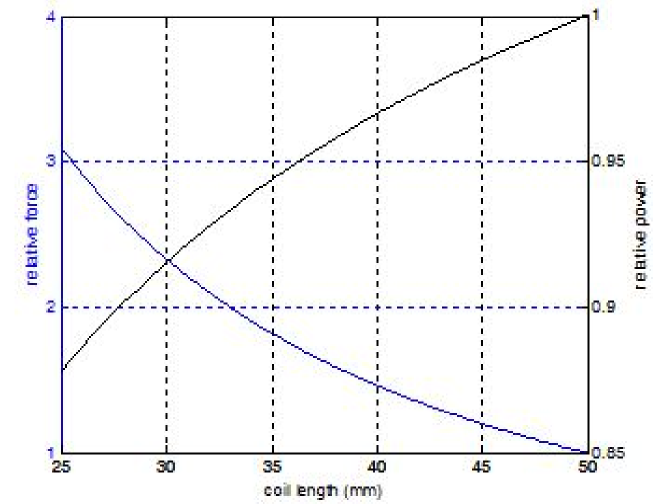

Figure 13. Relative force and power vs. l

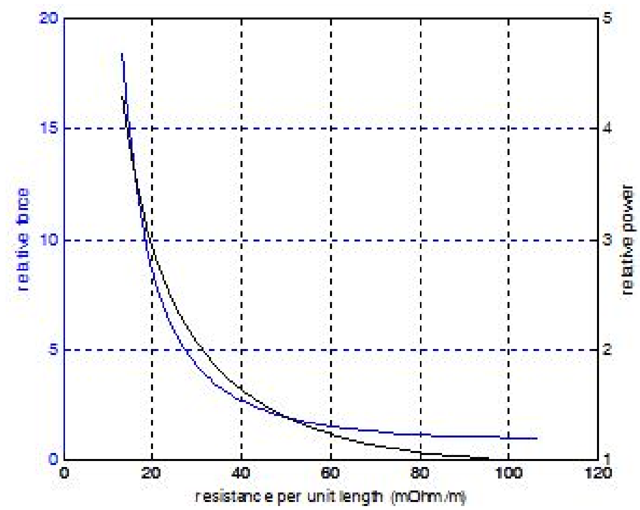

Figure 11. Relative force and power vs. γ

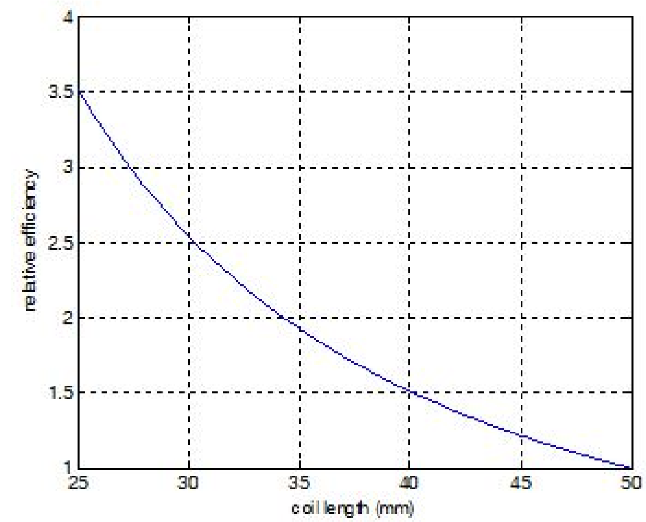

Figure 14. Relative efficiency vs. l

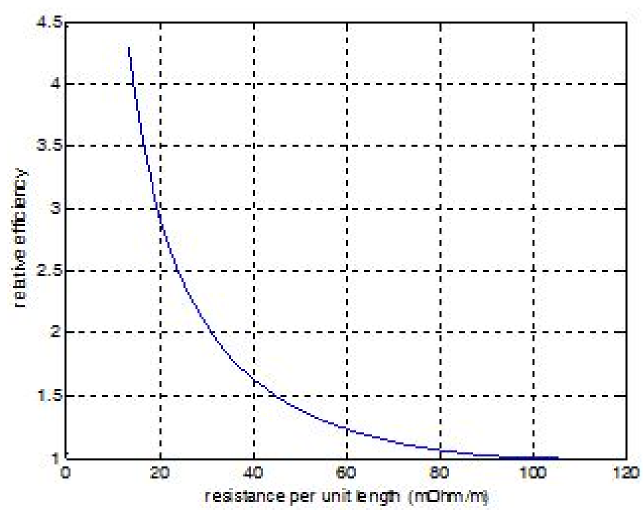

Figure 12, Relative efficiency vs. γ

Fig. 15 shows the measured force vs. turns from an actual solenoid using the same parameters as those simulated in Figs. 8, 9, and 10. The force was sampled 10 times at each of several decrements in N, and a least-squares approximation was used to find the best-fitting inverse quadratic in N, which is superimposed on the data. The results agree quite well with the simulation in Fig. 9. For example, decreasing the number of turns from 1503 to 700 increased the force by 126%. As in the simulation, this required twice the power. While this was a commercial solenoid, the coils were clearly hand-wrapped as they did not come off in uniform layers, which is the likely explanation for why the data does not span the best-fit curve at all samples of N.

If turns should not be a primary design factor for force, then how should one determine the nominal number of turns to use for a given application? Note that (22), in theory, allows the design of a coil for a given force with very few turns. In actuality, however, the limited current-carrying capacity of the wire will prevent the use of too few turns. For a DC coil, the steady-state current can be obtained from our estimate of total resistance in (18). We require that to be less than some limit based on the current-carrying capacity of the wire, which can be obtained from a table of wire gauges. A fractional safety factor is probably desired here, and some coil designers simply establish a rule-of-thumb for maximum current based on the cross-sectional area of the wire. Here we'll use the latter approach and call that parameter η (Amps per square meter of cross-section):

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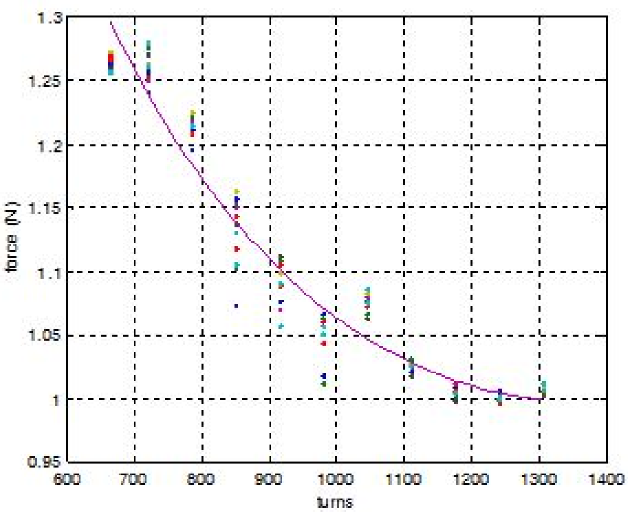

Figure 15. Force vs. turns for an actual solenoid

$$I_{DC} = \frac{V}{R} \leq \eta a^2 \tag{27}$$

Substituting (18) for R and solving (19) for a^2 results in:

$$\frac{V}{2\pi r_a \gamma N} \leq \eta \frac{\rho}{\gamma}$$
$$N r_a \geq \frac{V}{2\pi \rho \eta} \tag{28}$$

This provides a convenient lower limit for the product of the number of turns and the average coil radius. Notice also that γ has been cancelled out, although the resistivity of copper remains in the equation.

It has been suggested to the author that η = 3.5 A/mm^2 = 3.5 x 10^6 A/m^2 is a reasonable value, but the author knows of at least one arcade machine design that goes substantially over that value (and gets rather hot), although it remains below the recommended wire ampacity with insulation.

To achieve a desired force, a reasonable design sequence would be to first use (22), with the assumption of some reasonable winding factor, say 0.7, in order to determine an appropriate γ (wire resistance per unit length) and coil length, l. These two parameters can be traded off against each other such that a lower resistance wire could be used by lengthening the coil, or a shorter coil could be used by increasing the wire resistance. The number of turns, N, and coil radius, r_a (or r_0) could then be determined from (28). These two parameters can also be traded off against each other such the number of turns can be reduced with a corresponding increase in the radius, and vice-versa. Of course, the magnetic field and inductance approximations in (1) and (13) are based on long coils, so one would want to be wary of making a coil too short (l), in comparison to width (r_0 and r_a). These baseline parameter values could then be substituted into (25) which will include an estimate of the actual winding factor, and the parameters could then be tuned incrementally while ensuring that (28) remains satisfied.

Conclusion

This article explains the dependence of force for a cylindrical solenoid on coil length, coil radius, wire gauge, number of turns, packing density, and supply voltage using an analysis that goes well beyond that found in typical electromagnetic textbooks. The relationship between the number of terms and the solenoid force is validated against an actual solenoid.

The author's experience is that a significant number of engineers, typically exposed only to the relationship in (1), expect that increasing the number of turns will increase the force of a solenoid. Other engineers, realizing that it is usually voltage that is held constant, see that there will be a corresponding decrease in current and expect no change for a minor adjustment in turns on a baseline design. Most are thus surprised to learn that adding turns will in fact *decrease* the force. This article explains in detail how that comes about by revealing the effect that number of turns also has on coil radius.

A common explanation for why the removal of turns increases force is that it reduces wire length and resistance, and therefore increases current. (e.g., [7], [8]). This article shows that to be a misleading assessment of the situation. While adding (or removing) turns will indeed directly decrease (or increase) the current, this effect is counterbalanced by the fact that adding (or removing) a turn will also increase (or decrease) the magnetomotive force, as shown most simply in (1). Changing the number of turns thus affects the force only insofar as it results in a change to the average turn radius and consequently the winding factor. In addition, while force can be increased by reducing turns, that comes at a disproportionate cost in steady state power. This is a very inefficient way to increase force, as shown in Figs. 9 and 10. In contrast, decreasing the wire gauge or coil length will increase both the force and the efficiency simultaneously, as shown in Figs. 11 through 14.

Another factor that may contribute to confusion on this effect is that stronger solenoids taken off-the-shelf generally have fewer turns than weaker solenoids. The primary reason, however, is that they also have a thicker wire gauge, which leaves less room in a given space for turns on the coil. A wire with a larger cross-section has a lower resistance per unit length, γ. As shown in (22) and (25), this appears in the denominator of the force equation and thus leads to an increase in force. The mechanism, of course, is that the larger wire cross-section decreases resistance, which leads to a higher current for a given voltage. Unlike decreasing the number of turns, however, decreasing γ does not produce an inverse effect in the magnetomotive force. As a result, although not guaranteed, an off-the-shelf solenoid with fewer turns is likely to have more force than a solenoid with more turns.

Appendix A: Using Equation (8) Instead Of Equation (12)

Note that the inductance, L, shows up in the numerator of (12), but in the denominator of (8). If we were to repeat the

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development of (13) through (22) we would therefore find the force growing exponentially with *x*, rather than shrinking exponentially with *x*, which is precisely the opposite of what we find with real solenoids. This is a case where using idealized components with zero resistive losses leads to a completely incorrect result (perhaps not surprisingly, a similar conundrum can also be posed about charge transfer between ideal capacitors). It is possible to pursue a more realistic derivation from the AC point of view similar to (4), except including the wire resistance:

$$|I| = \frac{|V|}{|Z|} = \frac{|V|}{\sqrt{R^2 - (\omega L)^2}} \qquad \text{(A.1)}$$

Substituting this into (2), and dropping the magnitude signs yields:

$$W = \frac{1}{2} L I^2$$
$$= \frac{1}{2} \frac{V^2 L}{\left(R^2 - \omega^2 L^2\right)} \qquad \text{(A.2)}$$

In this expression L appears in both the numerator and the denominator. When ωL is small compared to R (x large, armature pulled out of the coil), then (A.2) is similar to (12), resulting in a force-stroke curve that falls with x. When ωL is large compared to R (x small, armature in), this equation behaves more like (8), with a factor of L in the denominator producing a force-stroke curve that rises with x. This behavior results in the possibility of a force-stroke curve with a local maximum somewhere mid-stroke. This is, in fact an often seen phenomenon in solenoids driven by AC, as pictured in Fig. A.1:

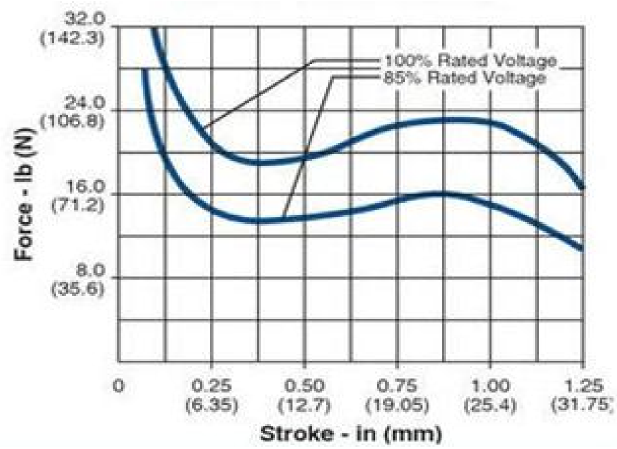

Figure A.1. Force Stroke Curve for an AC Solenoid

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