A Study of the Propagation, Refraction, Reflection, Interference and Diffraction of Ripple Waves

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A STUDY OF THE PROPAGATION, REFRACTION, REFLECTION, INTERFERENCE AND DIFFRACTION OF RIPPLE WAVES

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CONTENTS

PART I

HISTORICAL and THEORETICAL

I Introduction and Historical Sketch .......... 1
II Discussion of Theory of Ripple Waves ........ 3
III Summary .................................. 11

PART II

EXPERIMENTAL

I Object of This Investigation .................. 12
II Description of Apparatus and Method ........ 13
III Manipulation ................................ 16
IV Method of Obtaining Photographs .............. 17
V Ripples Stroboscopically at Rest .............. 17
VI Ripples Stroboscopically in Motion .......... 28
VII Other Experiments .......................... 29
VIII Discussion of Results ...................... 30
PART I

HISTORICAL and THEORETICAL
I INTRODUCTION AND HISTORICAL SKETCH

Introduction.-- Since the year 1678, when Huyghens pronounced the first wave theory of light, the history of physics shows a gradual broadening and extension of this theory to every branch of the science, and today we define a wave motion in the most general mathematical terms as being any function of x, y, and t, which will satisfy the differential equation:

$$\frac{D^2y}{DX^2} = \omega D^2y$$

meaning by this a disturbance that is periodic in both space and time. This definition includes not only the capillary and gravitational waves on liquids, but also applies to the vibrations in the ether known as heat, light and electrical waves, and to the sound waves in a gas.

A large amount of theoretical and experimental investigation has grown up around the subject of wave motion in the effort to determine the physical characteristics of such motion and also to explain, if possible, the phenomena of reflection, refraction, diffraction and interference. One of the most satisfactory experimental methods of attack has been found in the photography of ripple waves, especially since in this case we can visualize almost every phenomenon of wave motion. The object of this paper is to discuss the behavior of ripple waves and to describe experiments performed with them by the author to illustrate wave motion.

Historical Sketch.-- Historically the investigation of ripple waves began in 1874 with a theoretical discussion by Lord Kelvin, in

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* The partial derivative of p in respect to x is here written \( \frac{Dp}{dx} \) and this notation will be used throughout to distinguish from the total derivative \( \frac{dp}{dx} \).

1 Phil. Mag., 4, 42, 375.
which it was shown that the propagation of ripples depended upon surface tension. Matthiason\textsuperscript{2} tested the validity of Kelvin's formula, but because of the rough measurements of the ripple waves set up by a small pin point piercing a jet of water, failed to obtain a great degree of accuracy. Ahrendt\textsuperscript{3}, Riess\textsuperscript{3} and others made similar experiments, but it remained for Lord Rayleigh\textsuperscript{4} to develop the first accurate method of studying ripple waves. To make visible the extremely small disturbances in the plane of the liquid surface, he used a modified form of Foucault's optical method of testing plane surfaces. Furthermore he used the so-called stroboscopic method for making waves appear to stand still. Dorsey\textsuperscript{5} and Watson\textsuperscript{6} have extended and improved Rayleigh's method in making investigations on the surface tension of liquids.

Ripple waves have also been used to illustrate wave motion. Tyndall\textsuperscript{7} first made use of them for demonstrating the diffraction, reflection and refraction of wave disturbances. Later Vincent\textsuperscript{3} was able with more refined apparatus to obtain beautiful photographs of the same phenomena. The latter investigator generated the ripples on a mercury surface by means of a stylus attached to a vibrating tuning fork and illuminated them with an electric spark so that he could obtain instantaneous photographs of the waves.

\begin{itemize}
\item \textsuperscript{2} Wied Annalen, 33, 113.
\item \textsuperscript{3} Physical Review, Vol. 5, page 175.
\item \textsuperscript{4} Lord Rayleigh's Collected Works, Vol. III, page 283.
\item \textsuperscript{5} Physical Review, Vol. 5, page 176.
\item \textsuperscript{6} Physical Review, Vol. 12, page 237.
\item \textsuperscript{7} S. P. Thomson, Light Visible and Invisible, Chapter I.
\item \textsuperscript{8} Phil. Mag., Vol. 45, p. 413; Vol. 45, p. 191; Vol. 46, p. 293.
\end{itemize}
H. Schultze devised an electrical method for producing ripple waves, modifications of which have been made by A. H. Pfund and Palmer.

Waetzmann developed a method which he describes as follows. "For disturbance of the water, a fairly sharp stream of air is passed through a uniformly tapering tube against the water surface. The lighting takes place in the same equal intervals through a helium tube whose intensity varies in the period of the water disturbances. The tube is cut in two and a paper shield with one or more slits is inserted. The air stream can only enter when it finds an opening between the two tubes." A modified form of Waetzman's apparatus has been used by the author in the present investigation.

II DISCUSSION OF THE THEORY OF RIPPLES

The Equation of Continuity.- In the Eulerian method of studying the motion of fluids, attention is fixed upon a certain point \( P = (x, y, z, t) \). Suppose \( P \) (Fig. 1) to represent such a point surrounded by a small rectangular element \( dx \, dy \, dz \). The motion taking place therein can be expressed in the following way.

Let \( u, v, w \) represent the velocity components in the \( x, y, z \) directions at any time \( t \). Then the total flux of matter through the ends of the rectangle parallel to the \( yz \) plane is equal to the algebraic sum of that entering surface (a) and that leaving surface (b), which if \( \rho \) represents the density can be written as equal to

\[
\rho \, u \, dy \, dz - [\rho \, u \, dy \, dz + D \frac{\partial u}{\partial x} \, dx \, dy \, dz]
\]

9 Zeitschrift für Instrumen, p. 150, year 1906.
12 Zeitschrift für Physik und Chemie Unterricht.
In a similar way the other two terms may be obtained representing the total flux through the other two pairs of surfaces and thus, summing up all three terms we obtain the total change in quantity of matter taking place within the small volume. This change is, however, equal to the change in density with respect to time multiplied by the small element \( dx \, dy \, dz \). From these considerations it is evident that

\[
\frac{D\rho}{Dt} + \frac{D}{Dx}\rho u + \frac{D}{Dy}\rho v + \frac{D}{Dz}\rho w = 0.
\]  

(1)

which is the equation of continuity expressing the physical fact that there can be no creation or annihilation of matter within the volume \( dx \, dy \, dz \). A simplification is possible in case of practically incompressible fluids such as water, where \( \rho = \text{const.} \) and equation 1 reduces to

\[
\frac{Du}{Dx} + \frac{Dv}{Dy} + \frac{Dw}{Dz} = 0.
\]  

(2)

The Equation of Motion. - It remains now to study the equations of motion which can be set up when we have certain component forces \( X, Y, Z \) acting in the coordinate directions upon unit mass. Due to the change in pressure there will be a negative thrust in the \( x \) direction.

\[ 13 \text{ Any text on Hydrodynamics.} \]
equal to \(-\frac{\partial p}{\partial x} \, dy \, dx \, dz\), in addition to that due to the force \(X\).

From Newton's law that force equals mass times acceleration it is evident that the following equality must hold

\[
\rho dx \, dy \, dz \, \frac{du}{dt} = \rho X \, dx \, dy \, dz - \frac{\partial p}{\partial x} \, dx \, dy \, dz
\]
or

\[
\rho \frac{du}{dt} = \rho X - \frac{\partial p}{\partial x},
\]

similar expressions being obtained for the \(y\) and \(z\) directions.

The Velocity Potential.- If the motion is irrotational it can be shown that there exists some function \(\Theta\) such that

\[
\begin{align*}
    u &= - \frac{\partial \Theta}{\partial x} \\
    v &= - \frac{\partial \Theta}{\partial y} \\
    w &= - \frac{\partial \Theta}{\partial z}
\end{align*}
\]

and Lagrange\(^{15}\) has shown that if a perfect differential relation 
\[d\Theta = u \, dx + v \, dy + w \, dz\] exists at any moment and at any part of a fluid, it continues to do so. We may therefore use this new relation to modify the previous equations, for instance equation (1) becomes

\[
\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} + \frac{\partial^2 \Theta}{\partial z^2} = 0 = \nabla^2 \Theta
\]

The function \(\Theta\) is called the velocity potential since the derivative in respect to any direction gives the velocity in that direction.

Solution of equation of Motion: Remembering that if

\[
u = f(x,y,z,t)
\]

\[
\frac{du}{dt} = \frac{Du}{Dt} + \frac{Du}{dx} \frac{dx}{dt} + \frac{Du}{dy} \frac{dy}{dt} + \frac{Du}{dz} \frac{dz}{dt}
\]

then the equations of motion (3) may be written in the following form.

\[
\frac{Du}{Dt} + u \frac{Du}{dx} + v \frac{Du}{dy} + w \frac{Du}{dz} = X - \frac{1}{\rho} \frac{\partial p}{\partial x}
\]

\[
\frac{Dv}{Dt} + u \frac{Dv}{dx} + v \frac{Dv}{dy} + w \frac{Dv}{dz} = Y - \frac{1}{\rho} \frac{\partial p}{\partial y}
\]


Now suppose\(^{16}\) \(P = (x,y,z)\) represents an individual particle in a large mass of water, rotating about the z axis with an angular acceleration \(w'\). Fig. 2.

The vector \(R\) has the two components
\[
\begin{align*}
u &= X = R \cos \alpha = -R \sin (rx) = -yw'_z = (-R\frac{y}{r}) \\
v &= Y = R \cos \beta = R \cos (rx) = xw'_z = (R\frac{x}{r})
\end{align*}
\]
From which it is evident that
\[
\begin{align*}
\frac{Dv}{Dx} - \frac{Du}{Dy} &= 2w'_z = 0 \\
\frac{Du}{Dz} - \frac{Dw}{Dx} &= 2w'_y = 0 \\
\frac{Dw}{Dy} - \frac{Dv}{Dz} &= 2w'_x = 0
\end{align*}
\]
or
\[
\frac{Dv}{Dx} = \frac{Du}{Dy} \quad \text{etc.}
\]
when, and only when, the motion is \underline{irrotational}\(^{17}\), but since we have limited our discussion only to such a case as where there is no curl of the vector \(R\), therefore we may substitute the results of equation (6) in equation (5), at the same time putting \(q^2 = u^2 + v^2 + w^2\).


Furthermore let us presuppose X, Y, Z derivable from a force function independent of the time then, putting \( u = \frac{D\theta}{Dt} \) etc.

\[
- \frac{D}{Dx} \frac{D\theta}{Dt} + \frac{D}{Dx} (\frac{1}{2} q^2) = -\frac{D\Omega}{Dx} - \frac{1}{\rho} \frac{Dp}{Dx} \\
- \frac{D}{Dy} \frac{D\theta}{Dt} + \frac{D}{Dy} (\frac{1}{2} q^2) = -\frac{D\Omega}{Dy} - \frac{1}{\rho} \frac{Dp}{Dy} \\
- \frac{D}{Dz} \frac{D\theta}{Dt} + \frac{D}{Dz} (\frac{1}{2} q^2) = -\frac{D\Omega}{Dz} - \frac{1}{\rho} \frac{Dp}{Dz}
\]

(7)

Multiplying by \( dx, dy, dz \) respectively and adding, for any definite value of the time, \( t \).

\[
-d\left(\frac{D\theta}{Dt}\right) + d\left(\frac{1}{2} q^2\right) = -d\Omega - \frac{dp}{\rho}
\]

or

\[
\int \frac{dp}{\rho} = \frac{D\theta}{Dt} - \Omega - \frac{1}{2} q^2 + F(t).
\]

In so much as the resultant velocity \( q \) is a very small quantity its square may be neglected and as the function \( \theta \) is indeterminate to an additive function of \( t \), \( (u = - \frac{D\theta}{Dx} \) etc.\), we may suppose \( F(t) \) included in \( \frac{D\theta}{Dt} \) so that the equation now reads,

\[
\int \frac{dp}{\rho} = \frac{D\theta}{Dt} - \Omega
\]

(8)

Furthermore in the case of water the change of pressure is very slight so that, to a close approximation

\[
\int_{p_0}^{p_1} \frac{dp}{\rho} = \frac{\delta p}{\rho},
\]

also the function \( \Omega \) is merely gravitational, so we may write equation (8) in the form

\[
\frac{\delta p}{\rho} = \frac{D\theta}{Dt} - g z.
\]

(9)

If we consider only the motion in the \( xz \) plane and therefore on the \( z = 0 \) surface, the equation of continuity becomes,

\[
\frac{D^2\theta}{Dx^2} + \frac{D^2\theta}{Dz^2} = 0
\]

(10)

Then by means of the two boundary conditions, i.e., at the bottom of the liquid \( \frac{D\theta}{Dz} = 0 \), and at the surface, equation (9) must represent the
pressure, the solution of $X$ may be determined.

As a trial, let

$$0 = e^{ikx}(Ae^{kz} + Be^{-kz}). \quad (11)$$

From the first boundary condition, i.e., $\frac{d\theta}{dz} = 0$ where $z = 1$

$$\frac{d\theta}{dz} = ikx(Ae^{kz}) - e^{ikx}(Bk e^{-kz}) = 0.$$

or

$$Ae^{-k} = Be^{-k} = 2C = \text{const}.$$

Therefore to satisfy the first given boundary condition, i.e., $\frac{d\theta}{dz} = 0$ at the point where $z = 1$ we may write

$$\theta = C \cosh k(z - 1)e^{ikx} \quad (12)$$

If the motion be proportional to $e^{int}$ it will have a period equal to $\frac{2\pi}{n}$ as is seen from the periodicity of the function:

$$e^{int} = (\cos ut + i \sin nt) \quad (13)$$

Multiplying (7) by $e^{int}$ we have

$$\theta = C \cosh (z - 1) e^{i(kx + nt)} \quad (13)$$

which when expanded gives both real and imaginary parts, either of which satisfy the given equations, however, the solutions would give two systems of orthogonal curves. We may limit ourselves to one of these systems by choosing only the real part, so that (13) may be written in the form,

$$\theta = C \cosh k(z - 1) \cos(nt + kx) \quad (14)$$

Now if $h$ denotes the height of the water surface at a point $x$, and $T$ denotes the constant surface tension due to capillary action, then the pressure due to this force varies directly as $T$ and inversely to the

13 Lamb, "Dynamical Theory of Sound", page 53.


radius of curvature $R$ which may be expressed in the differential calculus by the equation

$$p = \frac{T}{R} = \frac{\sqrt{1 + \left(\frac{dz}{dx}\right)^2}}{\frac{d^2z}{dx^2}} T.$$ 

Since the waves are nearly flat $\frac{dz}{dx}$ may be neglected as compared with unity so that

$$p = -T \frac{d^2z}{dx^2} = -T \frac{d^2h}{dx^2}.$$ 

Substituting this value in equation (9) we have the second boundary condition.

$$\frac{T}{\rho} \frac{D^2h}{Dx^2} = g \frac{h}{h} + \frac{D\theta}{Dt}$$

Differentiating this equation in respect to $t$ and remembering that

$$\frac{Dh}{Dt} = -\frac{D\theta}{Dz} = w$$

or the $z$ component of the velocity, we have

$$\frac{T}{\rho} \frac{D^2\theta}{Dx^2Dz} = g \frac{D\theta}{Dz} - \frac{D^2\theta}{Dt^2}$$

(15)

which represents the final form of the second boundary condition.

Differentiating $\theta$ as defined by equation (14) and substituting in (15) it follows that

$$-\frac{T}{\rho} \left[ k^3 C \sinh k(z - 1) \cos(nt + kx) \right] =$$

$$g\left[ k C \sinh k(z - 1) \cos(nt + kx) \right] +$$

$$C n^2 \left[ \cos k(z - 1) \cos(nt + kx) \right].$$

Since we are dealing with the plane $z = 0$, the following simplifications are easily made so that

$$-\frac{kT}{\rho} = -g\left[ k C \sin h k l \cos(nt + kx) - Cn^2 \cosh k l \cos nt + kx \right]$$

or

$$\frac{n^2}{k^2} = \left( \frac{g}{k} + \frac{T}{\rho} \right) \tanh k l$$

(16)
In so much as the motion is proportional to $e^{ikx}$ and therefore to $(\cos kx + i \sin kx)$ it is evident that this determines the value of $k$ to be $\frac{2\pi}{\lambda}$, and since as has already been shown, the period of vibration equals $\frac{2\pi}{n}$, therefore

$$\sqrt{\frac{n^2}{k^2}} = \frac{\text{wave length}}{\text{period}} = \frac{\lambda}{n} = \text{Velocity of propagation.}$$

Therefore (16) can be written

$$V = \sqrt{\frac{\varepsilon \lambda}{2\pi} + \frac{2\pi T}{\rho \lambda}}$$

if only those vibrations are considered which have infinitely small vibrations so that $\tanh kl = 1$.

For any liquid of given surface tension $T$ it is seen from equation (17) that the velocity becomes the equal to the square root of the sum of two functions of the wave length, which themselves are constants, or we may write

$$V = \sqrt{f_1(\lambda) + f_2(\lambda)}.$$

The velocity will therefore have a minimum value when the sum $f_1(\lambda) + f_2(\lambda)$ is a minimum or as can be shown by calculus or algebra when $f_1(\lambda) = f_2(\lambda)$. Substituting the values in these functions and solving for the wave length it is found that

$$\lambda = 2\pi \sqrt{\frac{T}{\varepsilon}}$$

corresponds to the minimum value of $V$, and using this value to obtain the velocity we have

$$V = \sqrt{\frac{\varepsilon}{2}} \left( \frac{T\pi}{\rho} \right)^{\frac{1}{4}}.$$

For water, where $T = 75$ dynes the value of the minimum velocity is $23$ cm/sec. and corresponds to a wave length $\lambda = 1.7$ cm. The best idea of the relation of wave length to velocity can perhaps be gained

from the curve shown on opposite page.

Lord Kelvin has defined a ripple as the wave whose length is less than the wave length for which we obtain a minimum velocity and includes the dotted area under the curve, whereas he has defined a capillary ripple as one whose length is so small as to render negligible the first term \( \frac{g_\Lambda}{2\pi} \) in the value of the velocity squared.

III SUMMARY

Thus it has been shown that if the fluid is practically non-compressible, and nonviscous so that the forces remain conservative, and if the vibrations of the liquid have infinitesimal magnitudes, and if the motion is irrotational then the surface disturbance is representable to a close approximation by a sine curve. Therefore, if the above conditions are satisfied it is evident that ripple waves may be studied as examples of general wave motion. The experimental work in Part II has been built up on the basis of this conclusion.

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22 Watson's "Text Book of Physics."
PART II

EXPERIMENTAL
The object of this investigation is to study the properties of simple waves generated on a water surface, and in particular, to show how plane and circular waves are propagated, and how under certain conditions, they are reflected, refracted and diffracted. Some of these cases can be shown only by experiment since they have thus far proved to be too complicated for mathematical treatment.

II DESCRIPTION OF APPARATUS AND METHOD

The important experimental conditions to be fulfilled are first, to arrange a method for generating a pattern of ripples upon the water surface, and second, to make the waves visible. The waves were generated by allowing puffs of air to agitate a water surface. This was arranged by cutting the tube conveying the compressed air and inserting a disc with a circular row of equally spaced holes so that on rotating it would periodically interrupt the flow of air.

The waves were made visible by the stroboscopic method. Flashes of light were passed through the same set of holes that interrupted the air. Each wave as it progressed outward from the source was thus illuminated periodically, and since all the waves proceeded in the same way and were illuminated in the same positions they appeared to an observer as a single set of waves standing still, whereas really the visual impression was a composite of a large number of flashes of light.

The final form of the apparatus is shown in Fig. 3. The ripples were generated upon the surface of the water in the glass bottomed tank (A) which was supported by means of iron brackets (k1) from the wall of the laboratory, so that it would be free from the extraneous vibrations of the floor in the room. The waves were generated by the air puffs from the compressed air supply tube (d). This tube was made
Figure III

- Mirror (b)
- Wall Support
- Ground Glass Screen (a)
- Generating Tube (d)
- Water Surface in Glass Bottomed Tank (A)
- Adjustable Tube (e)
- Wall Bracket (k)
- Soft Rubber Tube (g)
- Lens (l)
- Tin Shield
- Arc (b)
- Rotating Wheel - Circular Row of Holes
- Sliding Block
- Variable Resistance (r)

Position of Camera

Ripple Pattern

Small Mirror (g)
D.C. Motor 4H.P.
of soft copper and was of about one mm. bore. Glass tubes varying in diameter from capillary dimensions to one cm. proved to be unsatisfactory since they could not be bent conveniently to give perpendicular incidence on the water surface. In order to secure a slow regulation of the height of the tube above the liquid surface, the adjustable tube (e) was designed, the details of which are shown in Fig. 4.

Fig. 4

One end of the tube (d) was soldered to a hollow brass cap which in turn fitted on to the head of the adjustable tube. When in place, the set screw in the cap would fit into the groove indicated in the diagram, thereby holding the generating tube stationary. The threaded adjustable tube could be worked up or down within the stationary brass cylinder (fastened to the edge of the tank) by means of the two nuts as shown in the figure, and therefore, the distance of the open end of (d) above the water could be adjusted at will. The connection for the air supply between the ripple tank and the rotating disc was made of soft rubber tubing so that extraneous vibrations would not be conducted to the water surface.

Any inconstancy in the frequency of the air puffs changed the wave pattern. To avoid this difficulty a large 1/6 H.P. direct curren
type motor was used and by having the interrupting disc of iron ten inches in diameter and one fourth inch in thickness the advantages of a fly wheel were obtained. All lost motion was eliminated by fastening the disc directly to the shaft of the motor. Holes one fourth inch in diameter were drilled in two concentric rows around the edge of the disc. At first each row contained over 20 holes, but it was found that a more constant rotation could be obtained by using only three and four holes and increasing the speed of the motor.

Light from the right angled arc (B) was brought to a focus on the same row of holes that the air passed through, but on the opposite edge of the disc. In order to secure an even illumination of the figure the arc lenses and mirrors were necessarily arranged so that their axes were in a straight line. One of the greatest sources of trouble was to secure a suitable reflecting surface at (j).

Ordinary mirrors proved unsatisfactory for this purpose since the heat from the arc would destroy the coating on the back of them, and would also melt the wax or glue with which they were held in place. Furthermore, the mirror which was necessarily of small area in order to get short quick flashed of light, was difficult to adjust. The best results were obtained by using a circular, polished aluminum surface cut at an angle of 45° as shown in fig. 5 below. The carbon arc was run on a 110 volt D.C. current and gave good illumination except for the colors explained by Mrs. Ayrton. These were troublesome, especially in taking photographs since the various colors acted differently upon the sensitive plates. At first some difficulty was encountered in keeping the light focused on the same row of holes when

1 Mrs. Ayrton, "The Electric Arc", Chap. I.
the disc was rotating rapidly. A piece of apparatus was finally designed as shown in fig. 5. In the sides of the U shaped bar two holes were bored so as to be in line with the small mirror j. The hole h, which faced the lamp was one fourth inch in diameter, while the other one was the same size as the mirror. When this bar was set astraddle a row of holes in the rotating disc it could be made fast by screwing the arm (H) to the table. The light could be easily focused on the stationary hole (h), even when the wheel itself was in motion, and the width of the sides of the bar being one half inch, a large amount of the extraneous light was shut out which added to the clearness of the reflected pattern. A 250 candle power solenoidal, incandescent carbon lamp was tried in place of the arc, however, it produced insufficient illumination and also gave an image of the filament. A lens L, (fig. 3) was interposed so that after reflection from the mirror (j) a parallel bundle of light rays would fall upon the bottom of the ripple tank. To secure sufficient air pressure the tube (i), fig. 3 was connected directly to the compressed air supply in the laboratory, and was regulated by means of a pinch screw cock. During all the experiments the room was darkened by heavy shades, and the extraneous light from the arc was shut off by the tin circular screen shown in diagram 3, or in the accompanying photograph No. 1 which pictures the apparatus as set up ready for use.

When photographs were taken, the camera was mounted on the
III MANIPULATION

The usual procedure after filling the tank to a depth of about one cm. with ordinary tap water strained through several thicknesses of cheese cloth, was to start the motor and then adjust the light until the surface of the water around the generating tube was evenly lighted. The air pressure was then turned on. Final adjustments were made by varying the position of the tube above the water, the speed of the motor, and the amount of air pressure, until a good pattern of ripples was produced. Every time the disc rotated the air pressure and the beam of light were cut off isoperiodically, and as the small ripples started out from the source they were illuminated at equal intervals and therefore the pattern on the ground glass screen appeared stationary. It was found that ripples of small amplitudes
could best be formed when the end of the generating tube just touched the liquid, but that for ripples of larger amplitudes it was more satisfactory to place the open end of the tube about one mm. above the surface.

IV METHOD OF OBTAINING PHOTOGRAPHS

Photographs were taken with a 4 x 5 camera, using, with few exceptions, the largest aperture, No. 3, which allowed the time of exposure to be shortened without destroying good definition. Cramer's "Crown Emulsion" and "Double Coated" dry plates were used and developed in an Ortol solution the formula of which is given below.

Solution A
(Water 15 ounces
(Ortol 120 grains

Solution B
(Water 15 ounces
(Sodium Carbonate 1 ounce
(Sodium Sulphite 1 ounce

For Developer
(One ounce each of solutions A and B

Fixing Bath
(Water 4 ounces
(Hypo 1 ounce
(Sodium Bisulphite 1/4 ounce

V RIPPLES STROBOSCOPICALLY AT REST

It has already been shown how it is possible to secure a pattern of ripple waves which are apparently at rest. This arrangement allowed investigations to be made in a number of type cases that are illustrated in the following paragraphs.

Propagation: Photographs Nos. 2 and 3 show the constancy of the wave pattern and therefore the regularity of the propagation of the ripples upon the water surface. No. 2 was taken with a single point source of waves and No. 3 with a two point source to be described later. The camera was placed about three feet in front of the ground.
Photograph No. 4

Photograph No. 5

Photograph No. 6

Photograph 7
glass screen and the smallest stop (No. 128) was used. Exposure was made for 25 minutes. The frequency of the air puffs was observed to be over 80 per second, so that from a simple calculation it is evident that in both cases approximately 100,000 ripple waves were generated within that time. The fact that the crests and troughs of so many waves coincided so closely as is shown in the photographs indicates the constancy of the disturbance on the liquid surface. The dark line extending toward the center of the pictures is the image of the generating tube which in this case was lowered so as just to touch the water. The photographs Nos. 4, 5, 6, 7, show the wave pattern set up by a single source. In each case the speed of the motor was varied so as to obtain a slightly different wave length. These last photographs were taken with a No. 3 stop and exposures varying from 4 to 10 seconds.

Reflection: In general, the reflecting surfaces were made of either brass or copper, and in all cases were cleaned with HCl before being placed in the water. It was found necessary to submerge the reflecting surfaces so that a thin film of water would cover them so that the incident ripple waves would not pass over but would be reflected. Otherwise, the water could raise along the edge of the reflecting surface and produce distortion of both the ripples and the transmitted light.

Photograph No. 7 shows the reflected wave pattern from a straight edge made of a uniform brass bar whose end dimensions were 1 cm. x 1/2 cm. The largest stop (No. 3) on the camera was used, and an exposure of 4 seconds was made. The Lloyd's bands of interference are also clearly shown extending off at equal angles from the straight edge.
Photograph No. 9

Photograph No. 9 illustrates the result of reflection from a corrugated surface as is shown in the diagram below, Fig. VI, and consists of a half circle cut from a brass cylinder which was 12 cm. in
diameter. The notches were carefully machined along the inner surface and were 3 mm. wide and 3 mm. deep. The generating tube was placed at S and in order to secure a reflected wave pattern of sufficient amplitude it was necessary to increase the air pressure so much that the center of the pattern became distorted, however, the complicated interference effects are clearly seen near the edge of the reflecting surface in the picture. Stop No. 16 was used with an exposure of 10 minutes, whereas, the length of the ripple waves in this case was slightly greater than the width of each corrugation. By varying the length of the waves many different and interesting patterns were formed, which could be studied at will. These are interesting because thus far they have proven too complicated for mathematical treatment.

This method of investigation was extended to circular, elliptical and parabolical surfaces. These were made of narrow copper sheets, and submerged in the water so that reflected wave systems could be produced within their boundaries. In this way, for example, it could be shown that the ripples generated at one focus of an ellipse appear to converge to the other focus. Photographs of these patterns were not taken.

**Refraction:** By placing a drop of acid (H Cl) or light oil on the surface of the water near the generating tube, the ripple pattern could be made to show the refractive influence of the second liquid. The drop itself would gradually move over the surface of the water so that no photograph could be obtained by a time exposure. It could also be shown that ripples in very shallow water (1 mm. or less)

travel slower than those in deeper water. This was done by placing a piece of plate glass in the bottom of the tank so that the waves would travel through the very thin layer of liquid above the submerged surface.

Interference: Photographs Nos. 10 and 11 show the interference pattern set up by a two point source. To secure these pictures two generating tubes were used instead of one. These were soldered to the same cap and then fastened over the head of the adjustable tube, as shown in Fig. IV. The current of air thus divided and a double set of isoperiodic air puffs, having the same phase, were obtained.

Photograph No. 10
Photograph No. 11
Perpendicular to the line joining the two point sources in No. 10 a light line is visible. On either side of this is a system of confocal hyperbolas having the two sources as their foci. These lines
mark the region of least disturbance, being the locus of points where a crest from one source coincides with a trough from the other source. Photograph No. 11 shows the results of spreading the generating tubes further apart. Between the two centers of the system appear a system of ellipses. The system of hyperbolas in this case were produced outside of the region between the two sources but were too indistinct to show in the picture. By changing the wave length, a resulting shift in the interference bands could be observed.

**Diffraction:** According to Huyghens' principle a plane wave front incident upon an opening that is narrow as compared to the length of the wave gives rise to a circular wave front on the opposite side which appears to diverge from the opening as shown in Fig. VIIa.

[Diagram of a circular wave front (a) and straight lines (b)]

When the opening A is made much larger the propagation takes place as shown in Fig. VIIb and can be explained by a combination of Huyghen's wave theory and Fresnel's theory of interference. Several methods were used to show these phenomena. In the experiments performed, the incident wave front was circular instead of plane, but

3 Barton, Text Book of Sound, page 72-80.
the same reasoning applies as given above and shows that in this case the curvature of the incident wave will, or will not, be changed according to the width of the opening.

Photographs Nos. 12 and 13 were taken before the final form of the apparatus had been adopted. The air puffs were allowed to impinge upon the water surface between two wooden blocks which extended above the water and appear as the dark rectangles in the picture. At first sight these photographs seem to show nicely what would be expected from the theory, but since the bars were not immersed in the water, the surface film rising along their edges distorted the waves as they left the opening. This action of a film changing the apparent wave length of the ripples was shown by Dorsey. This difficulty was overcome as in the previous experiments by barely submerging the bars. Photographs Nos. 14 and 15 show the result of single openings but of different widths, whereas No. 16 shows the interference phenomena produced by a large number of openings which therefore resembled a diffraction grating.

Photographs Nos. 17 and 18 were taken with the aid of the apparatus shown in Fig. VIII. A thin strip of brass was bent into the indicated form and was fastened by a screw at (k) so that the width of the opening could be adjusted at will.

![Fig. VIII](image-url)
The generating tube was placed at 6, and the level of the water was adjusted so that the waves were allowed to pass out only through the aperture A. In No. 17, the curvature of the emergent wave is very different from that of the original wave. In No. 18, however, the opening is much larger and the ripples are seen to advance with but little change in curvature. These photographs were taken with the large stop and from 4 to 8 seconds exposure.

VI RIPPLES STROBOSCOPICALLY IN MOTION

In some of the work it was advantageous to be able to view the actual progress of the ripple waves. This could not be done by the unaided eye since the waves were too small and moved too quickly. The stroboscopic method, however, allowed observations to be made whereby the waves could be seen apparently to move forward as slowly as desired. This was done in the following way. The light was put on a row of three holes and the air on one of four. Since there was not the same number of holes in the two rows the periods of the air puffs and of the illuminations differed by a certain constant ratio. For this reason the waves were not always in the same phase when lighted, and therefore they moved stroboscopically along the surface.

This apparent motion was very slow so that any particular wave could be observed during all of its stages of progress. This gave a very interesting method for studying reflection and diffraction. Furthermore, it showed vividly how the flow of energy takes place in a system of interference fringes, as for example, that set up by a two point source as in photograph 10 where we have a pattern of confocal hyperbolas.

It is evident that complete interference does not take place along the light lines shown in the figure, since enough energy must
cross them to give rise to the waves between the fringes. This phenomena has been discussed by Wood\(^5\), who suggests that the wave disturbance should travel out between the lines of interference as between two silver walls. Due to reflection first from one wall and then from the other, the wave front should first appear with a curvature as though coming from one source and then should appear to come from the other. When the ripples were allowed to advance stroboscopically, this progress of the waves between the interference fringes could be observed and studied under many different conditions.

VII OTHER EXPERIMENTS

Other wave phenomena may be shown with the aid of ripple waves. For instance, in moving the air puffs rapidly across the surface of the water, the wave length in the front of the moving source varied in accordance with Doppler's principle.

In general the surface of the liquid acts as a stretched membrane, which has its own frequency of vibration, and only when the frequency of the air puffs is approximately the same, will the pattern of ripples appear to be uniform. This condition was secured by adjustment of the air pressure and the rotation of the disc, but if, instead of an impulsive pressure such as the air puffs, we could arrange a disturbing force which would vary constantly and yet periodically, the liquid surface would soon lose its own period of vibration and take up that of the external force. As a result of this forced vibration the frequency of the liquid surface itself would be eliminated.

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The apparatus used in this experiment is very well adapted for the purpose of demonstrating various wave phenomena, since we can show stroboscopically stationary or moving wave patterns. To be able to show not only a standing wave, but also to be able to witness the actual progress of a single wave or group of waves during reflection and diffraction is very instructive. Furthermore, the experiment suggests a method for investigating more in detail the reflection from complicated surfaces and the nature of diffraction under various conditions, both of which are interesting from the standpoint of acoustics. With slight modification the apparatus could be used for illustrating various phenomena of wave motion to large audiences.

In conclusion I wish to thank Dr. F. R. Watson for his many helpful suggestions and for his inspiration in the work, and Professor A. P. Carman, for the facilities placed at my disposal.